Lagrangian Analysis with Dynamic Systems Theory

14th NAVMSMO Technical Interchange Meeting

ONR Code 322PO

August 19, 2004

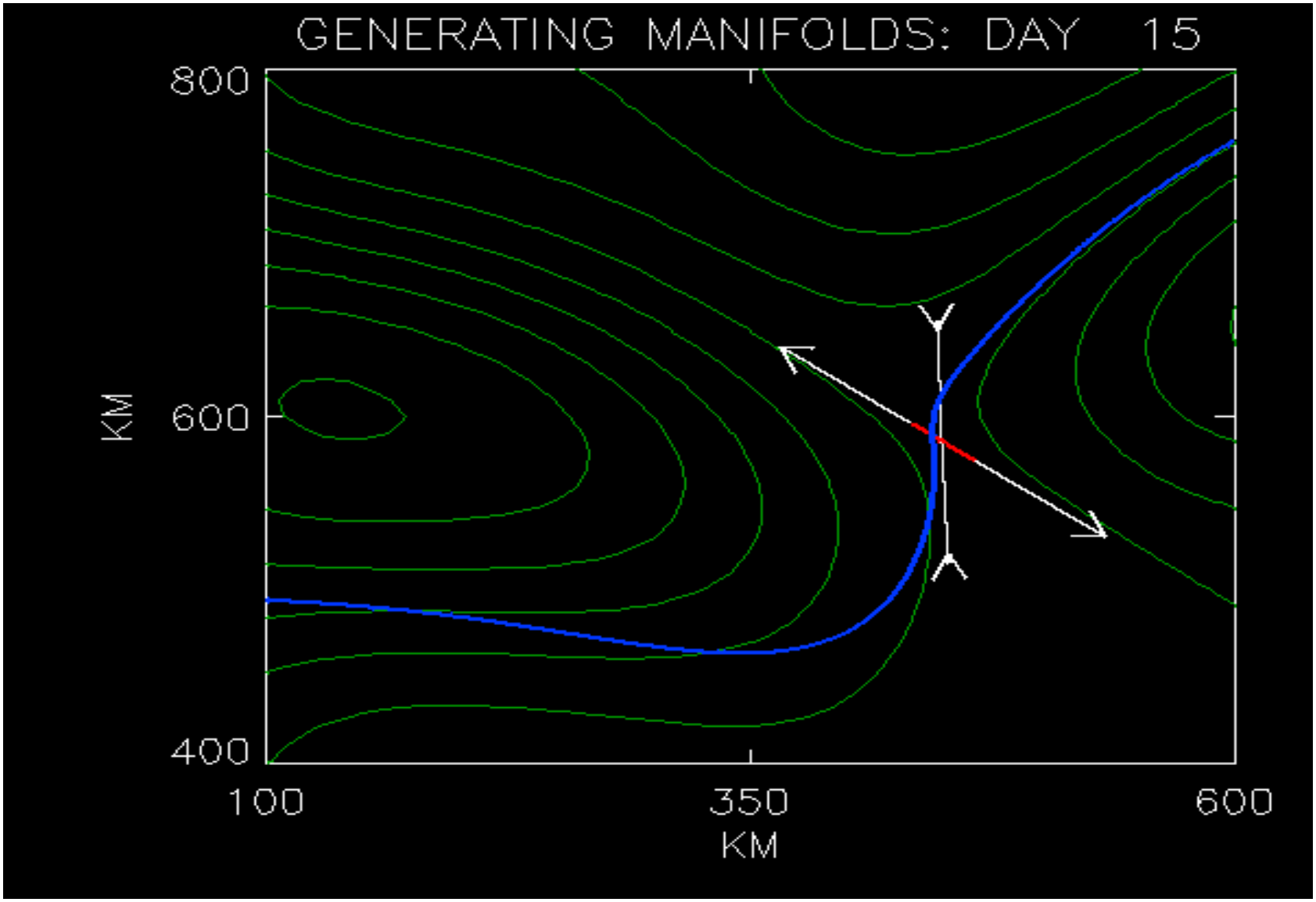
Dynamic Systems Concepts

- Hyperbolic trajectories
- Convergent (inflow) and divergent (outflow) manifolds
- Lagrangian Coherent features
- Direct Lyapunov Exponent Analysis
- Synoptic Lagrangian Maps

Approach

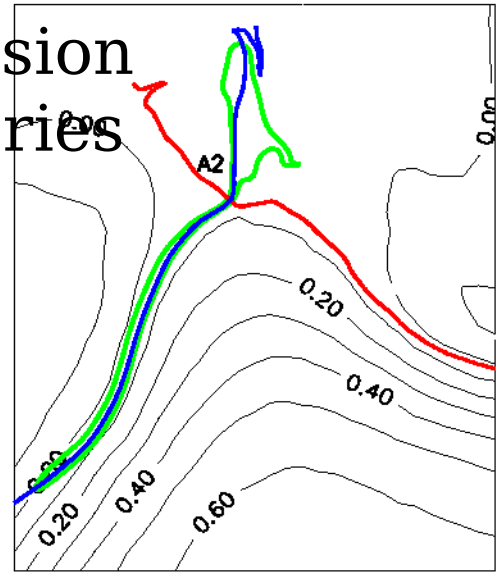
Dynamic Systems Theory applications in oceanography

- More than 10 years of collaboration between mathematicians and oceanographers
- Studies of mixing
- Lagrangian prediction
- ONR initiative

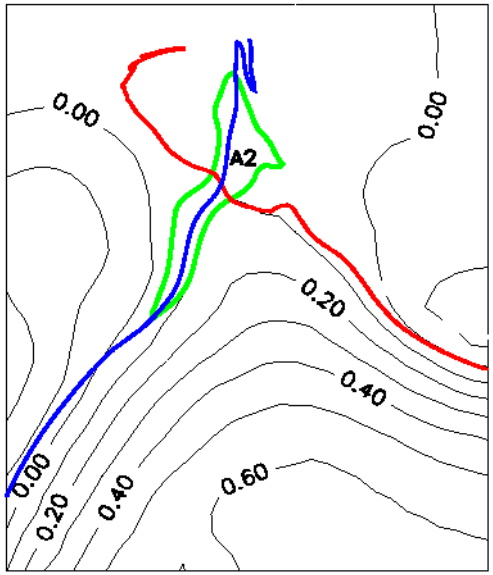

GENERATING MANIFOLDS: DAY 15
800
600
400
KM
100
350
600
KM

Stretching and compression near hyperbolic trajectories

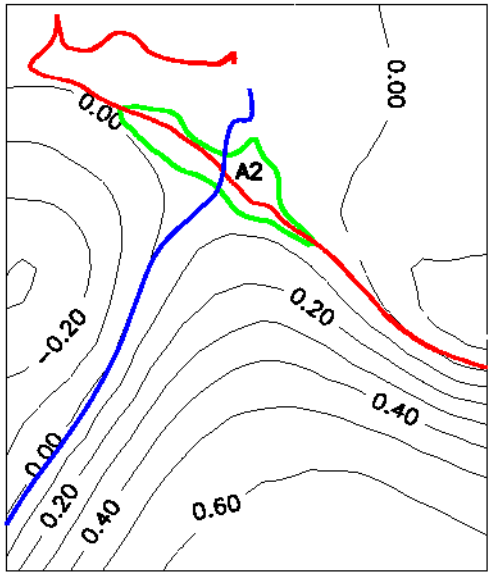
April 14



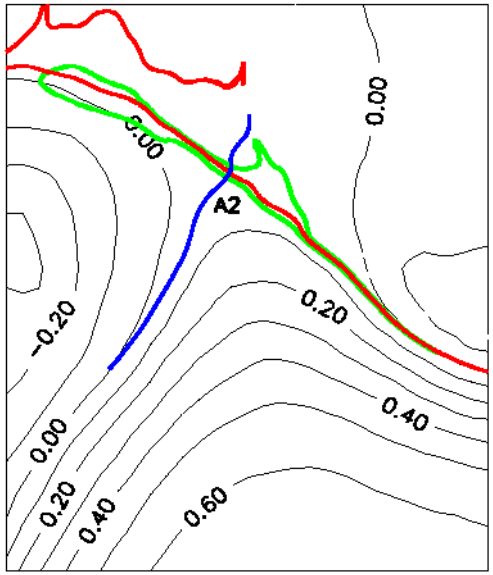
April 15



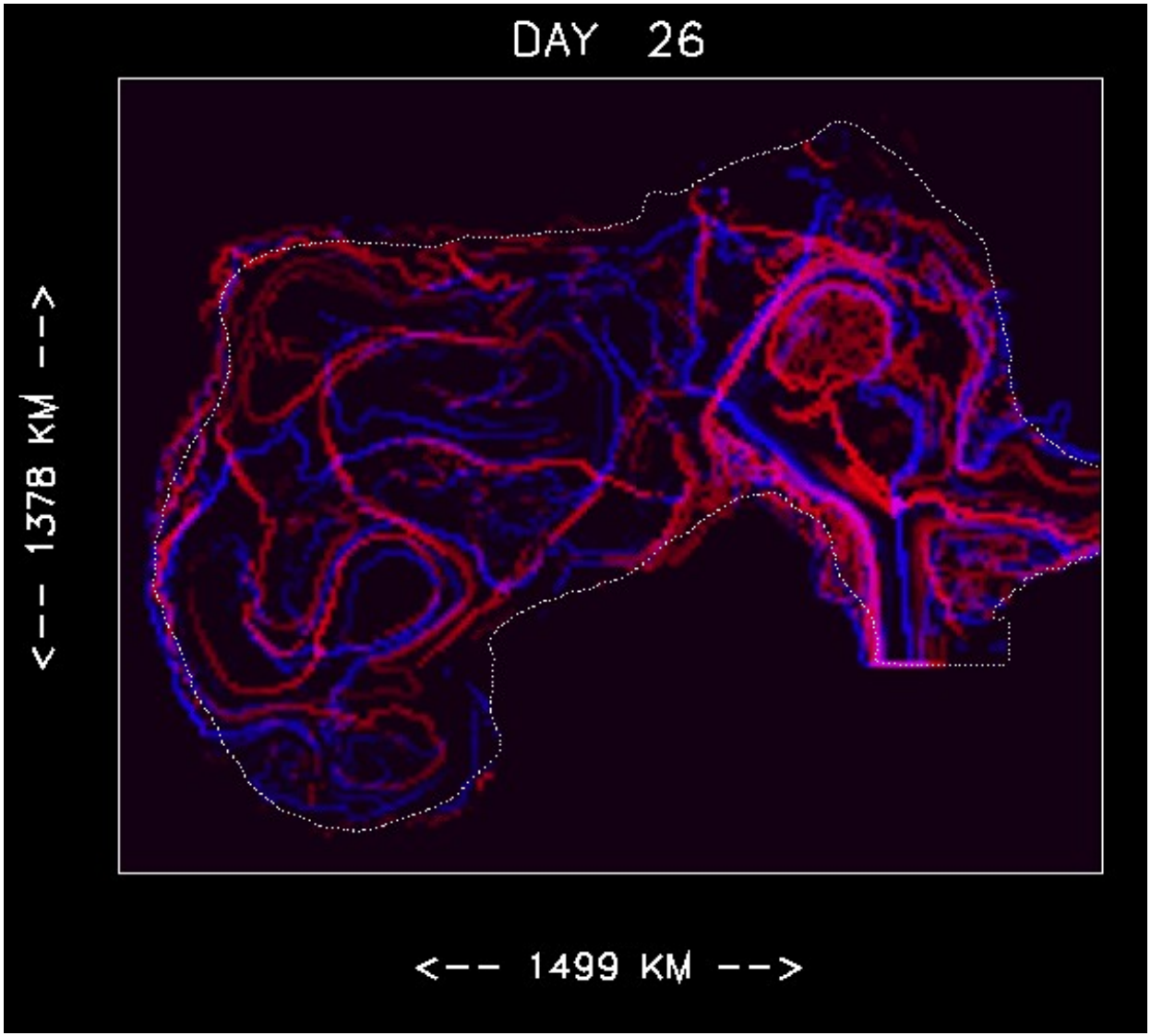
April 17



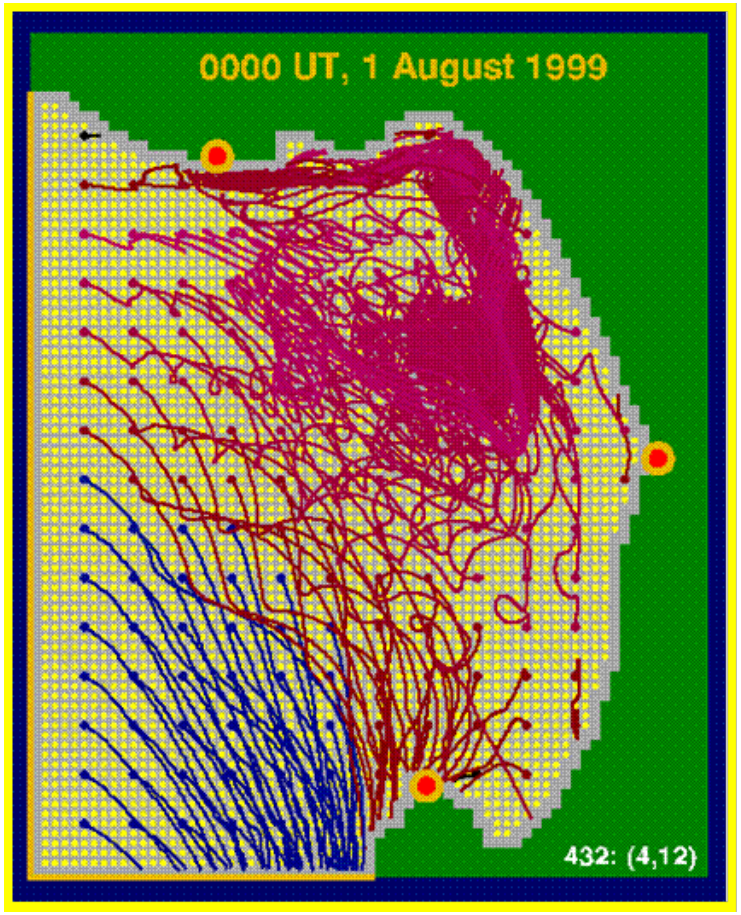
April 18

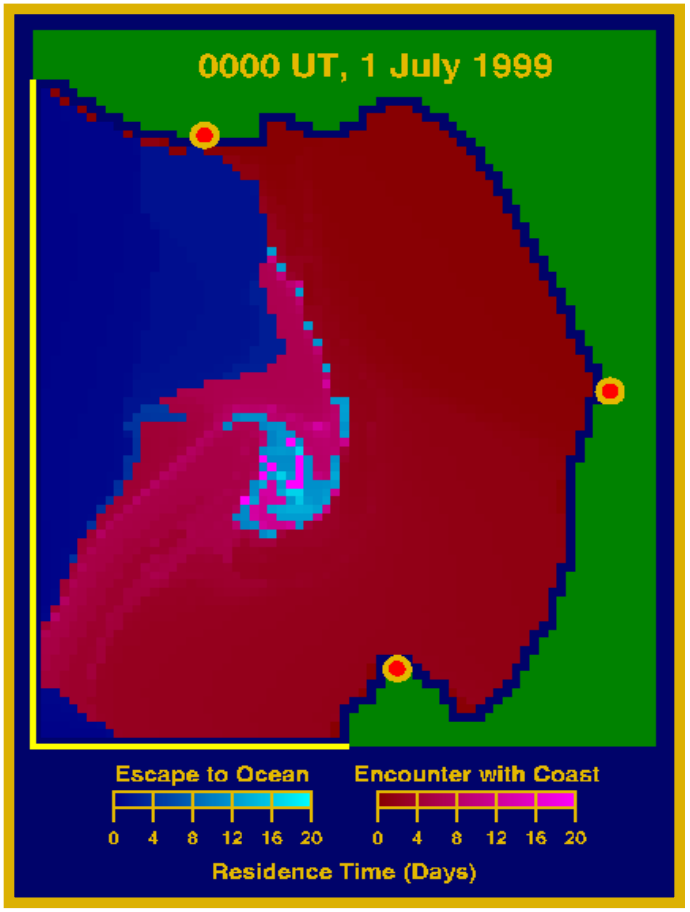


Direct Lyapunov Exponent Analysis

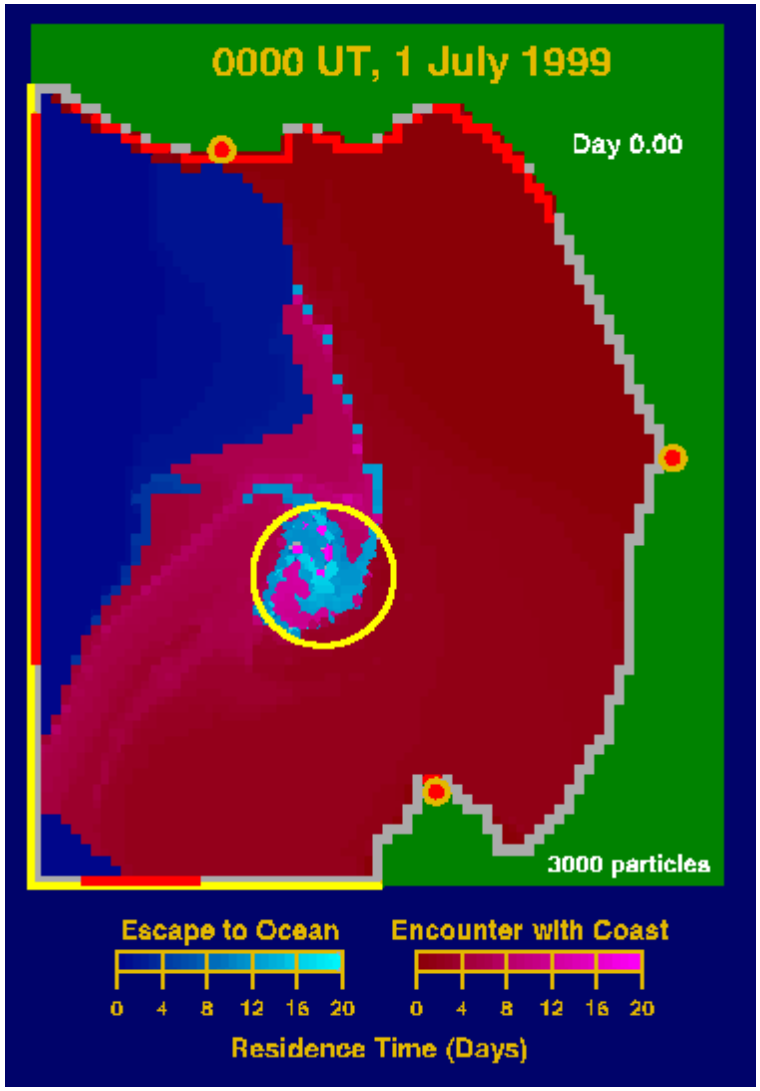


Synoptic Lagrangian Map





Fate and residence time of particles in the Monterey Bay



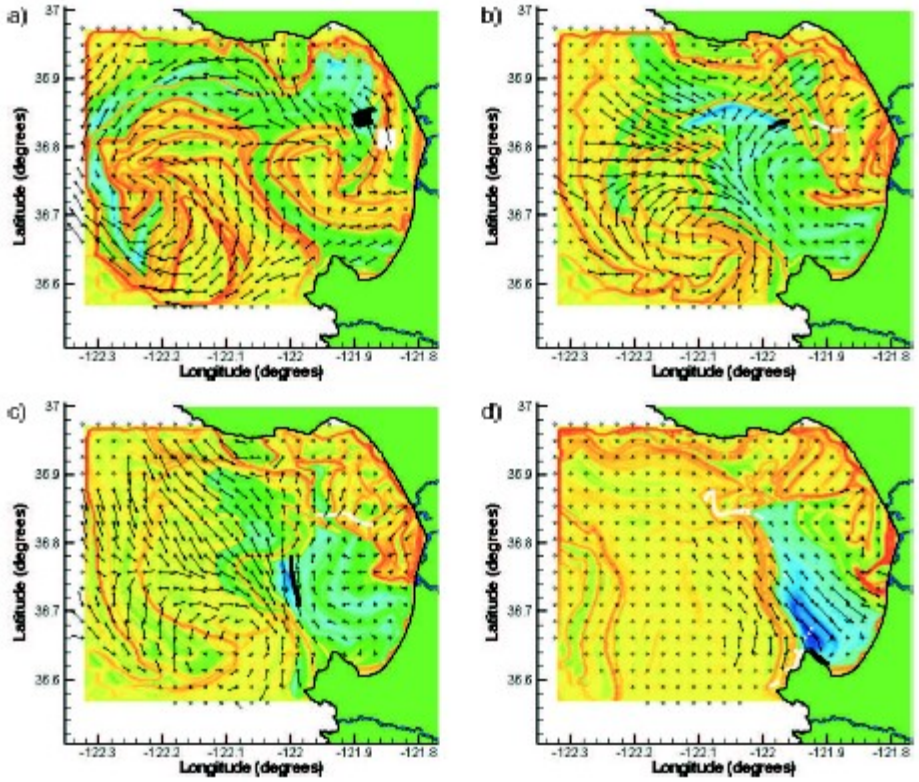
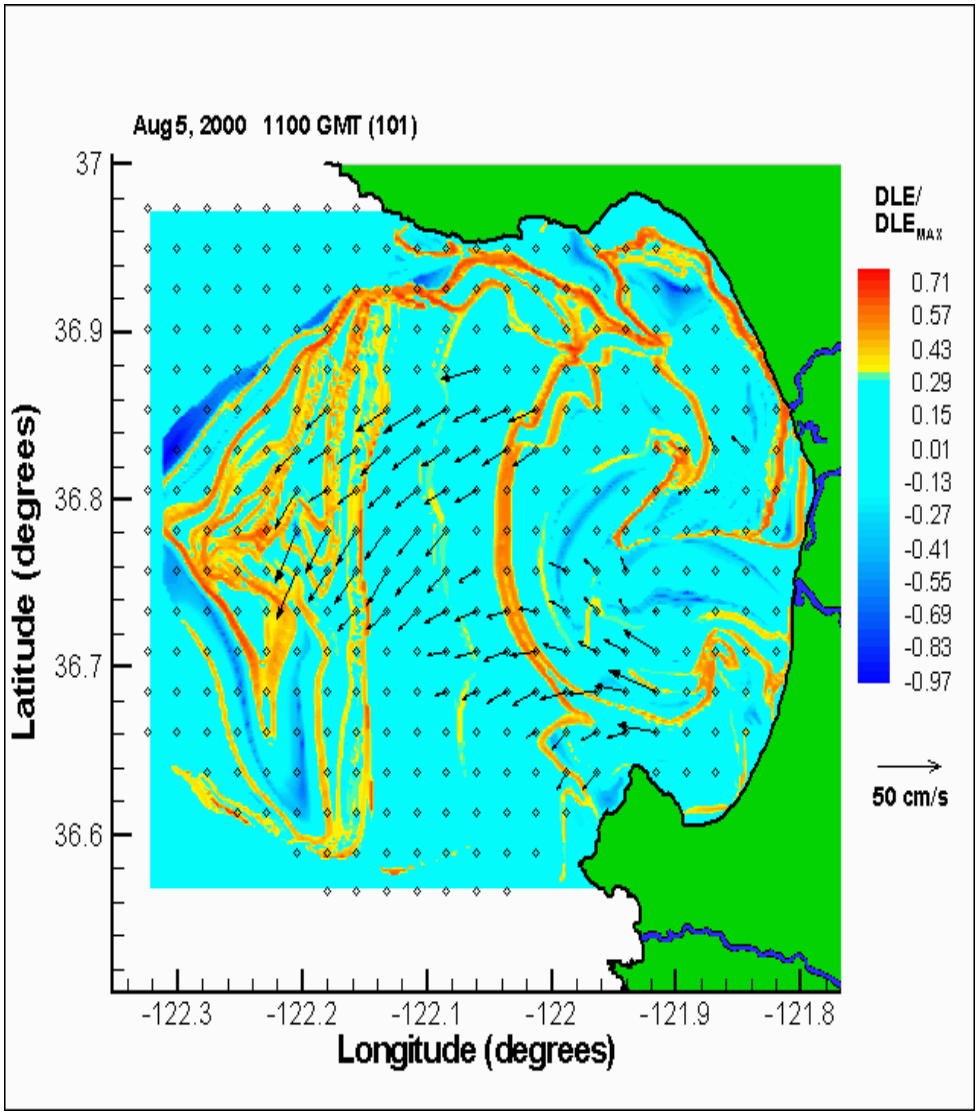


FIG. 1: Two parcels of contaminants released from the same position near Moss Landing at 22:00 GMT, August 6, 2000 (black) and at 09:00 GMT, August 7, 2000 (white). The black parcel remains in the bay, while the white parcel departs from the bay. The black arrows show instantaneous surface velocities captured by the HF radars. The ridges of the DLE field reveal the hidden Lagrangian structure of the bay at the same time instants. The black and white parcels are released on different sides of the barrier line, thus explaining the different behaviors of their trajectories.


Aug 5, 2000 1100 GMT (101)
37
36.9
36.8
36.7
36.6
Latitude (degrees)
-122.3
-122.2
-122.1
-122
-121.9
-121.8
Longitude (degrees)
DLE/
DLE_MAX
0.71
0.57
0.43
0.29
0.15
0.01
-0.13
-0.27
-0.41
-0.55
-0.69
-0.83
-0.97
50 cm/s

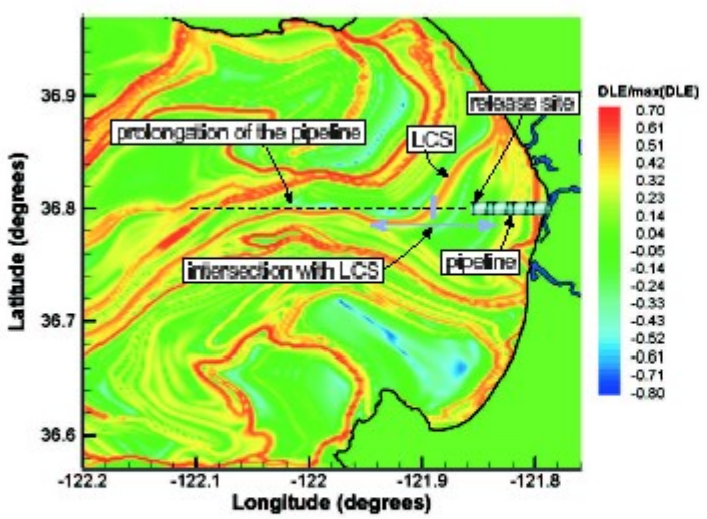


FIG. 2: A pipeline carries contaminants to be released in the bay from the Moss Landing area. Also shown is the barrier point, i.e., the instantaneous intersection point between the Lagrangian coherent structure (LCS) and the axis of the pipeline.

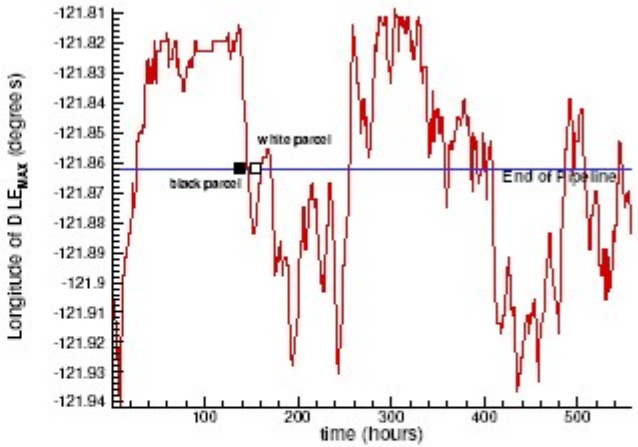


FIG. 3: Oscillation of the barrier point along the axis of the pipeline. The zero reference time corresponds to 07:00 GMT, August 1, 2000. The horizontal line marks the location of the outlet of the pipeline. The black and white squares represent the release time and release longitude of the parcels featured in Fig. 1.

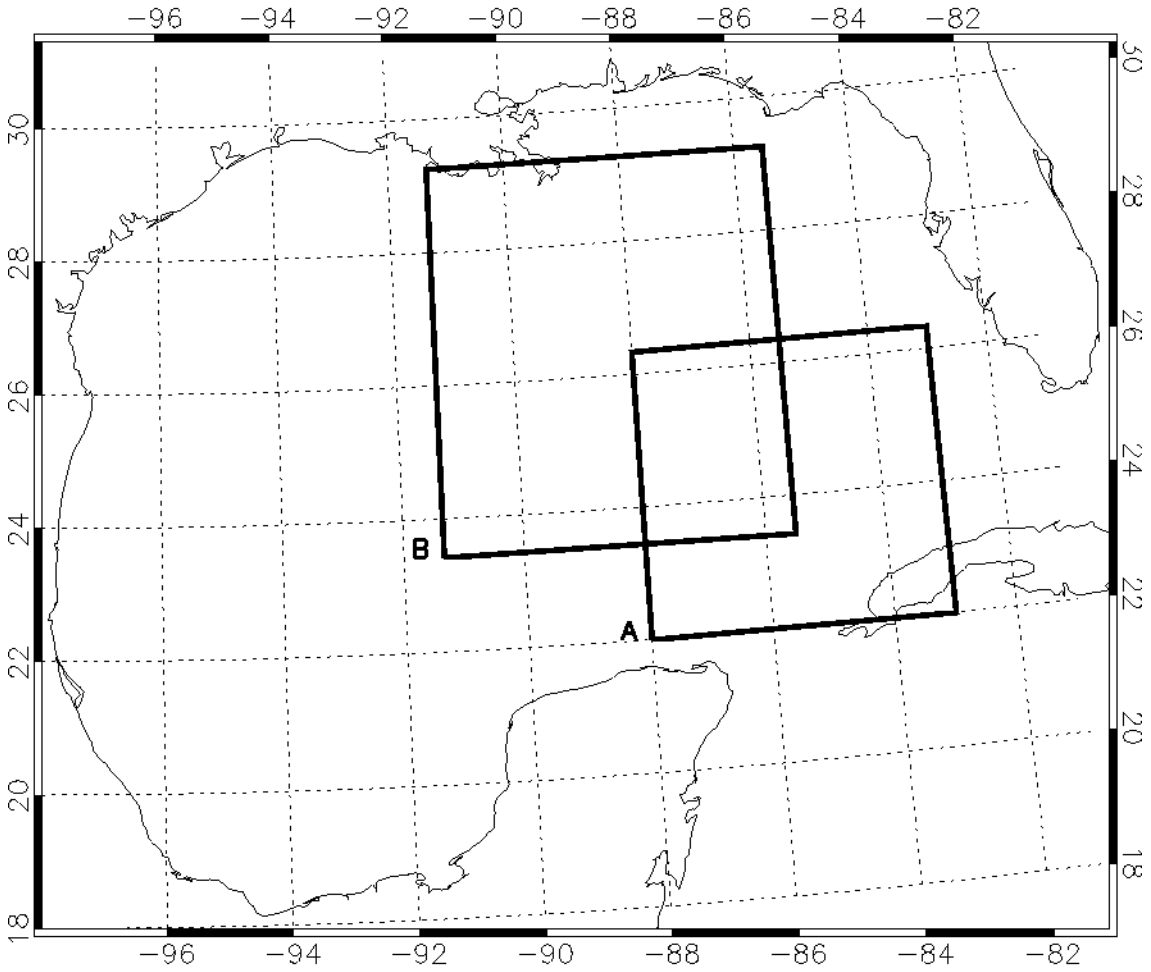
Chlorophyll dispersal by eddy-eddy interactions in the Gulf of Mexico

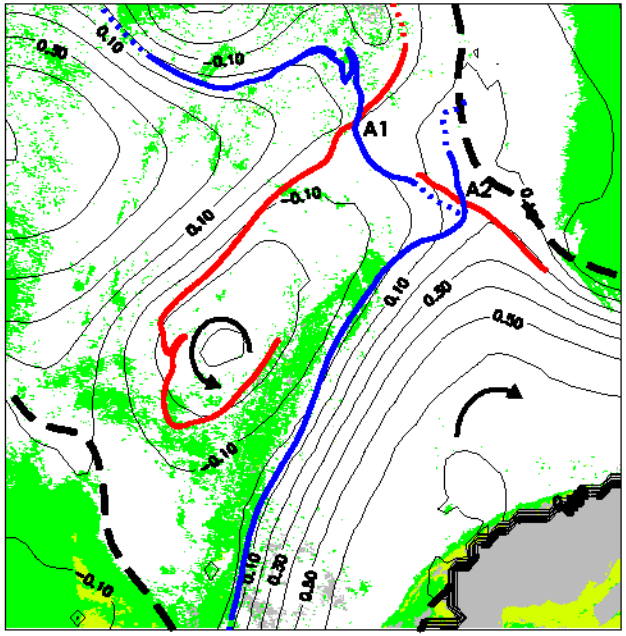
by

Toner, M., A.D. Kirwan, Jr., A.C. Poje, L. Kantha,
F.E. Muller-Karger, and C.K.R.T. Jones

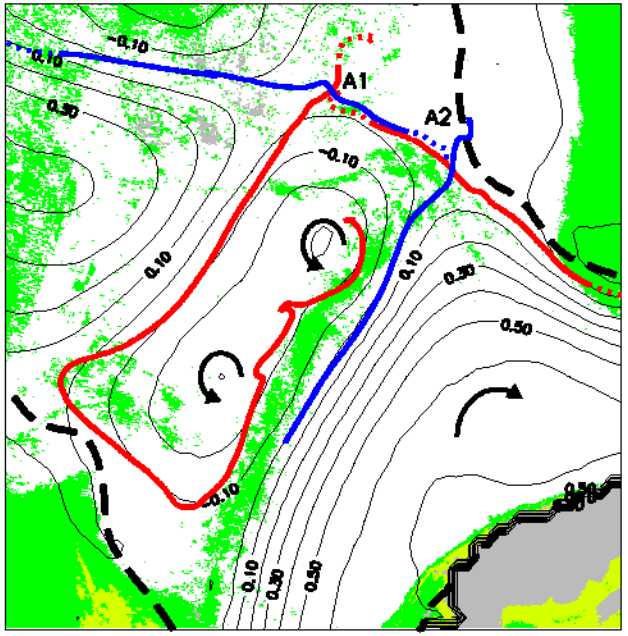
Journal of Geophysical Research, in press, 2002

The Gulf of Mexico





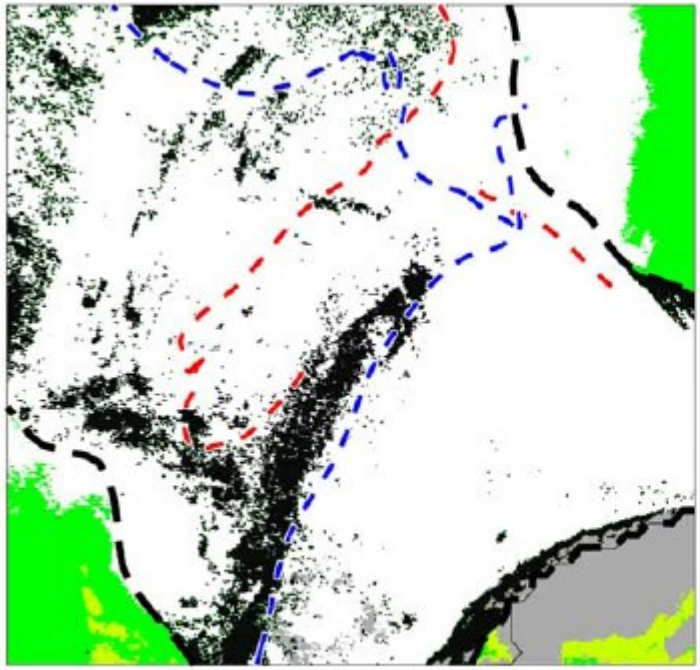
April 12, 1998



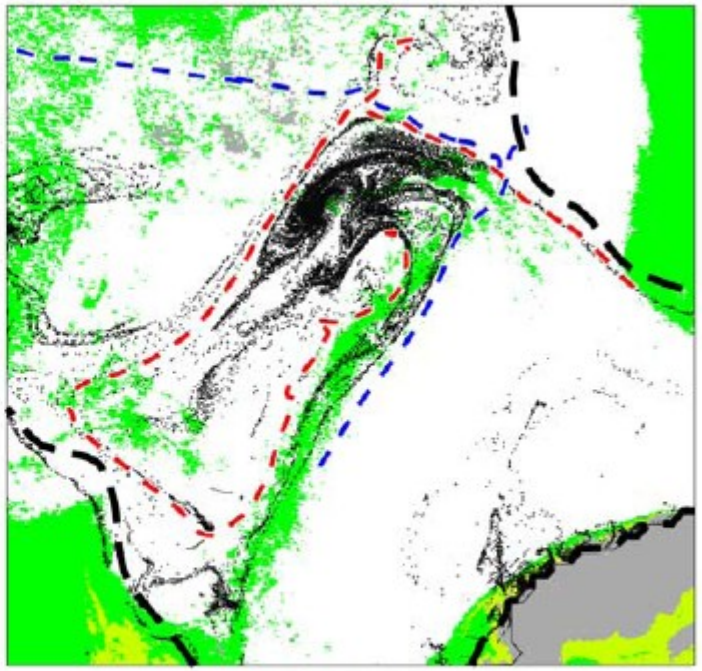
April 17, 1998

Region A - cyclone adjacent to the Loop Current

Inter-eddy advection documented by ocean color and material curves from CUPOM.



(a) Region A: April 12



(b) Region A: April 17

Numerical plume

- Trajectories initialized at chl-a pixel locations
- Advected with 50m velocity

Letter Report, NRL 7140-98-002

AN/WSQ-6 Navy Drifting Buoy Program Testing in the Littoral Warfare Advanced Development (LWAD) 98-2 Sea Test

by

NRL Code 7183

and

NRL Code 7140

21 May 1998

WSQ Buoy Drift LWAD 98-2

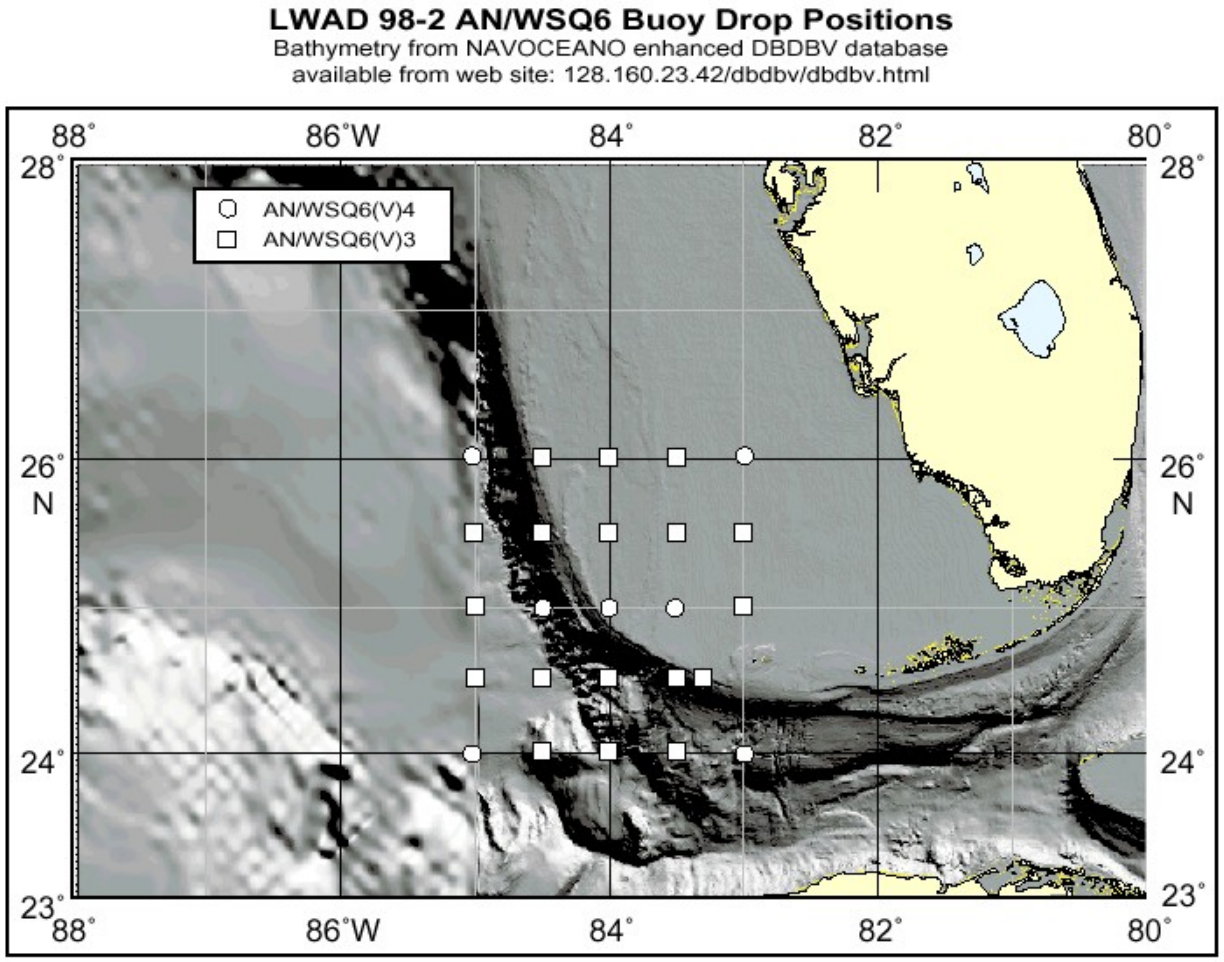


Figure 2. Test and Evaluation Area During LWAD 98-2 AN/WSQ-6(V)3 and AN/WSQ-6(V)4 Tests.

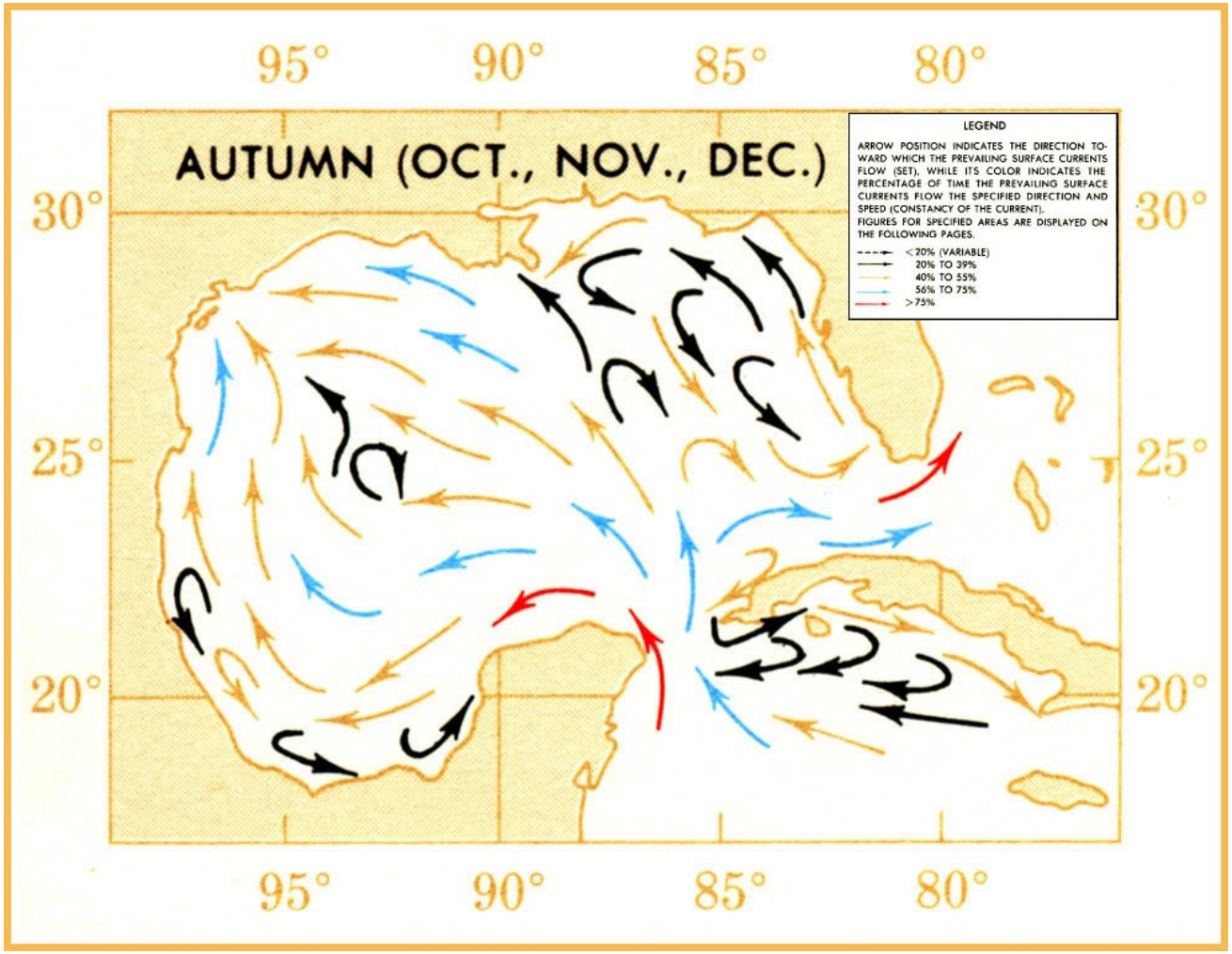
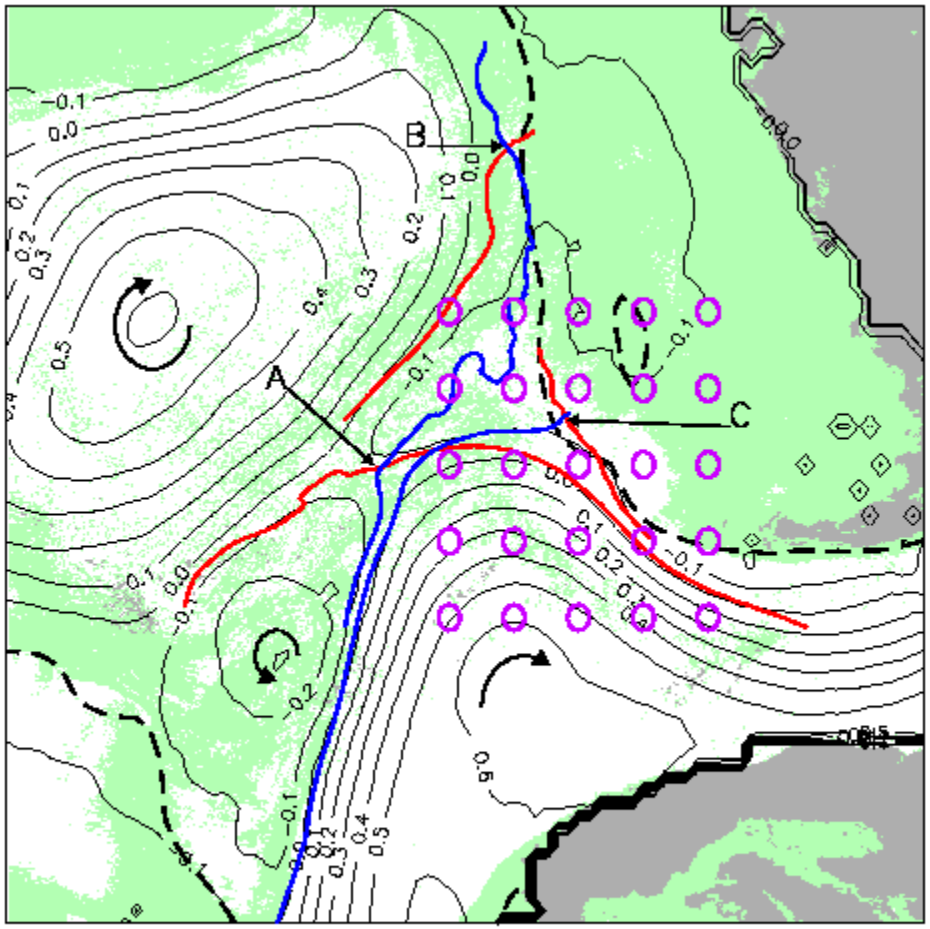
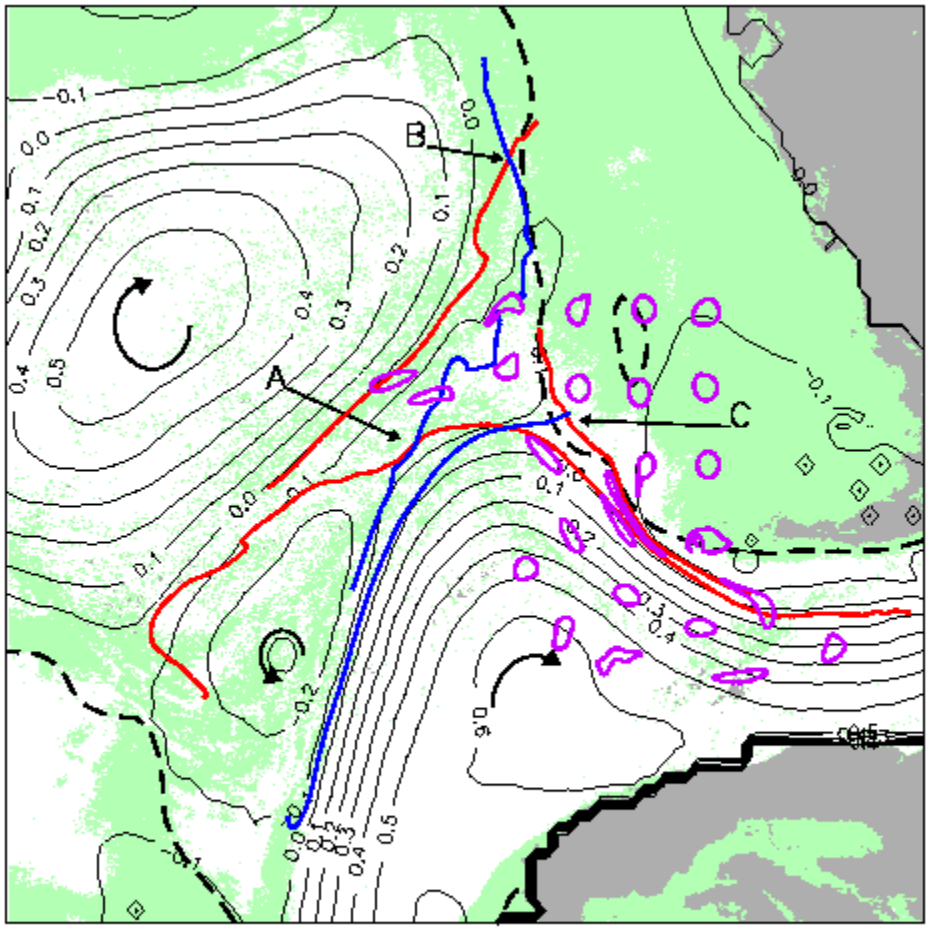

AUTUMN (OCT., NOV., DEC.)
LEGEND
ARROW POSITION INDICATES THE DIRECTION TOWARD WHICH THE PREVAILING SURFACE CURRENTS FLOW (SET), WHILE ITS COLOR INDICATES THE PERCENTAGE OF TIME THE PREVAILING SURFACE CURRENTS FLOW THE SPECIFIED DIRECTION AND SPEED (CONSTANCY OF THE CURRENT).
FIGURES FOR SPECIFIED AREAS ARE DISPLAYED ON THE FOLLOWING PAGES.
<20% (VARIABLE)
20% TO 39%
40% TO 55%
56% TO 75%
>75%
95°
90°
85°
80°
30°
25°
20°

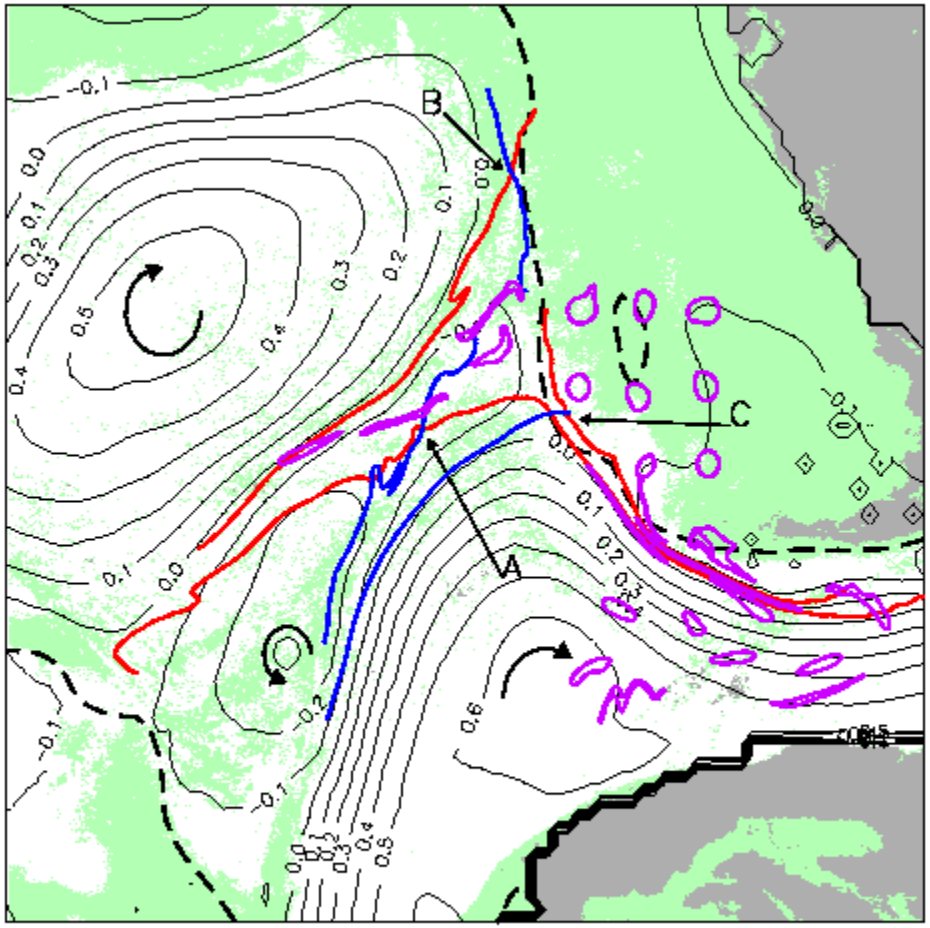
Figure 5. Climatological flow pattern of the Gulf of Mexico for Autumn, source states that "in the Gulf of Mexico, current speeds do not vary significantly between autumn and winter." Mean current speeds for the area of interest are between 0.9 and 2.5 kts. Arrows indicate direction toward which the prevailing surface currents flow. Arrow color indicates percentage of time the prevailing surface currents flow in the specified direction. (NAVAIR, 1974).

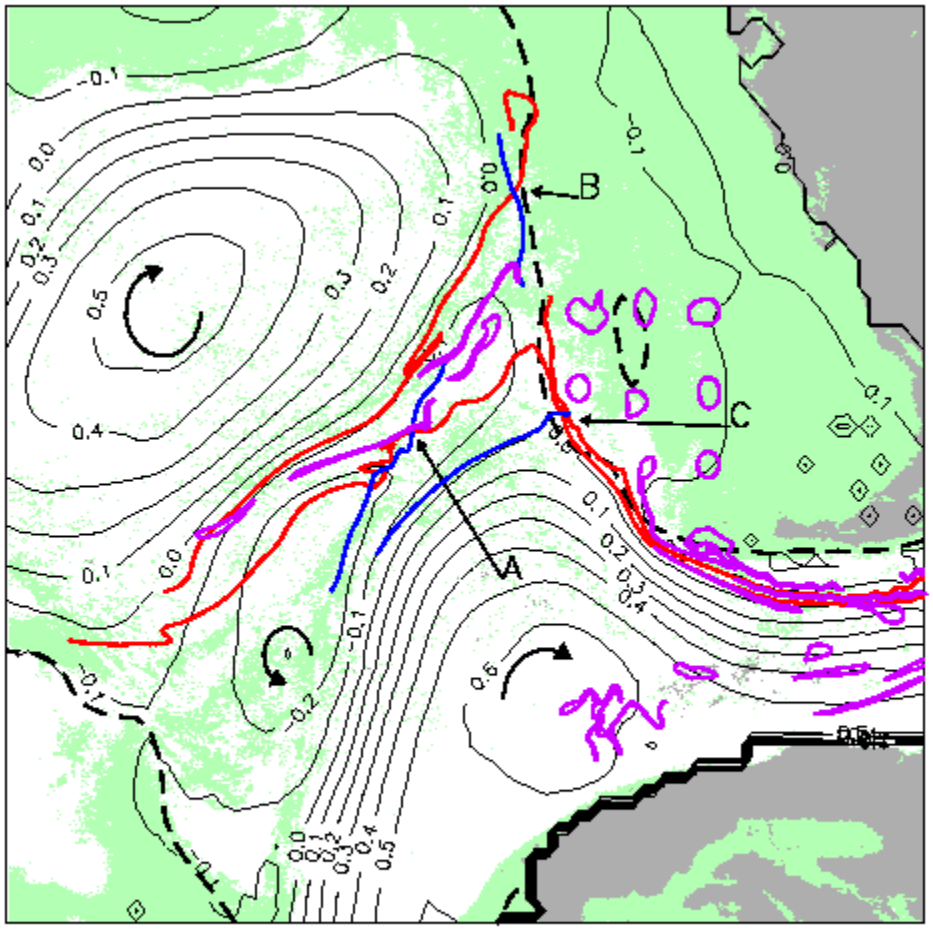
Lagrangian Analysis

- Parse the Gulf into distinct regions based on their advective fate with inflowing and outflowing manifolds
- Evolution of material discs centered at drifter deployment locations
- SeaWiFS ocean color data used as “tracer of convenience”

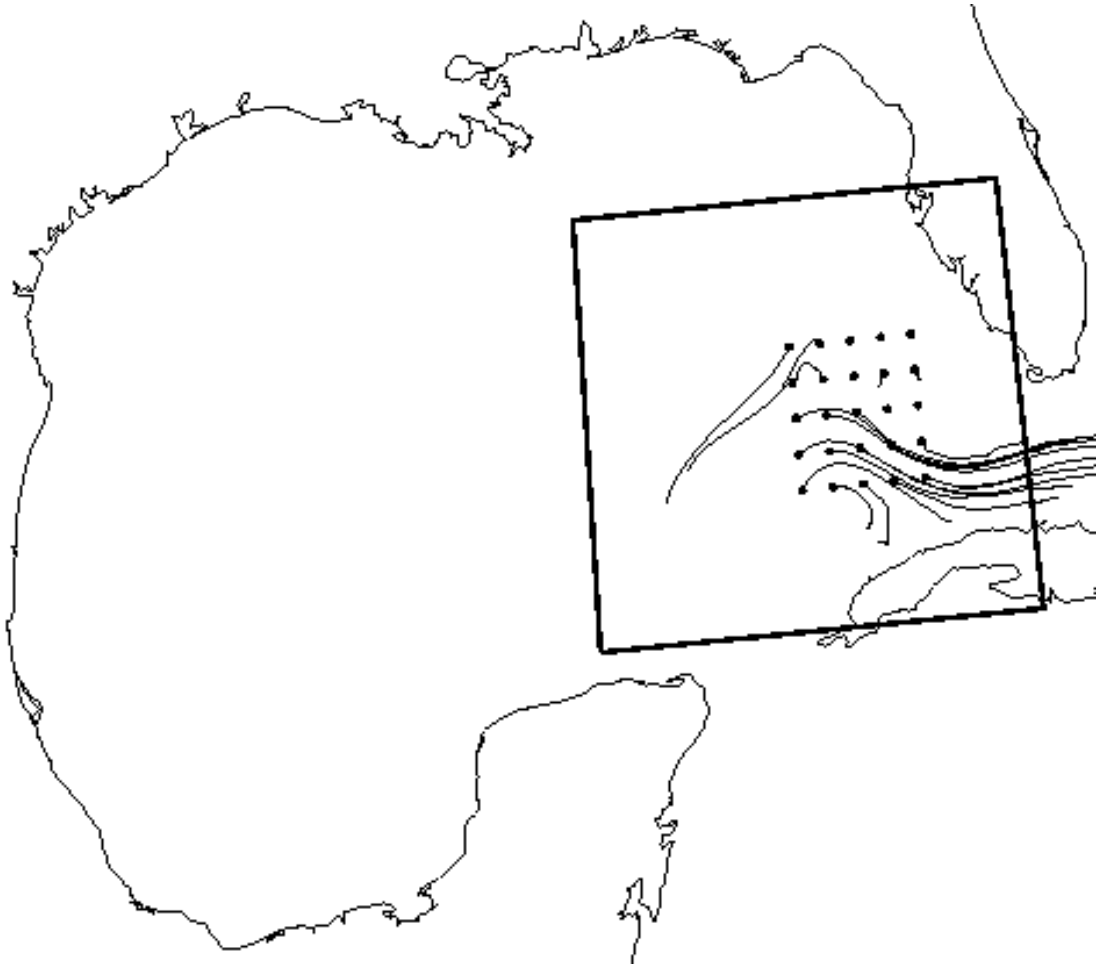

A
B
C
OUTFLOWING
INFLOWING
>0.25
<0.25
NO DATA
9 KM RADIUS
FEB 26


A
B
C
OUTFLOWING
INFLOWING
>0.25
<0.25
NO DATA
9 KM RADIUS
FEB 27


B
A
C
OUTFLOWING
INFLOWING
>0.25
<0.25
NO DATA
9 KM RADIUS
FEB 28


A
B
C
OUTFLOWING
INFLOWING
>0.25
<0.25
NO DATA
9 KM RADIUS
FEB 29

Five day trajectories initialized at sensor deployment



WSQ Buoy Drift LWAD 98-2

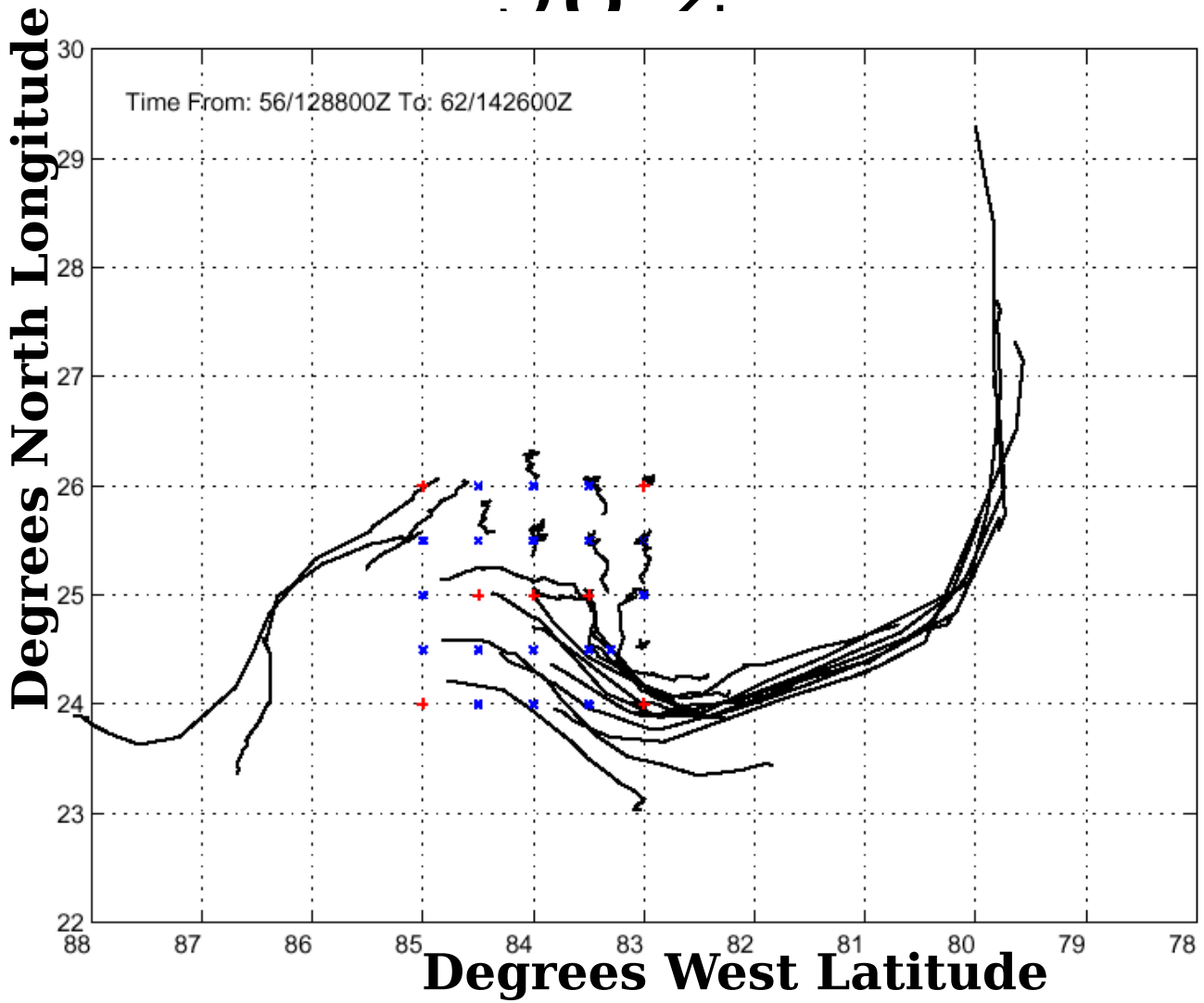
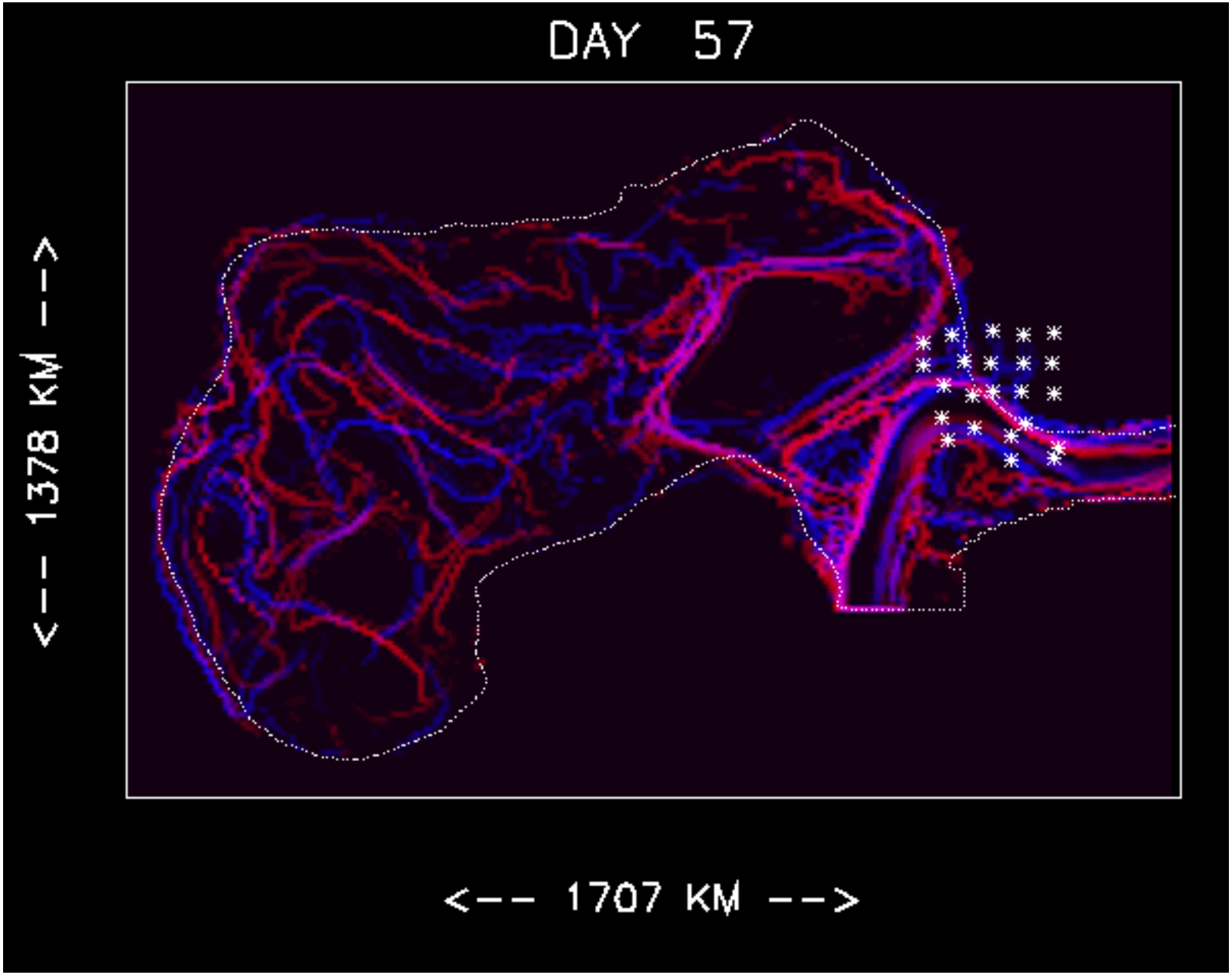


Figure 3. Buoy drift patterns for exercise period (February 25-March 2, 1998). An x indicates AN/WSQ(V)3 drop location, a + indicates AN/WSQ(V) 4 drop location.


DAY 57
<-- 1378 KM -->
<-- 1707 KM -->

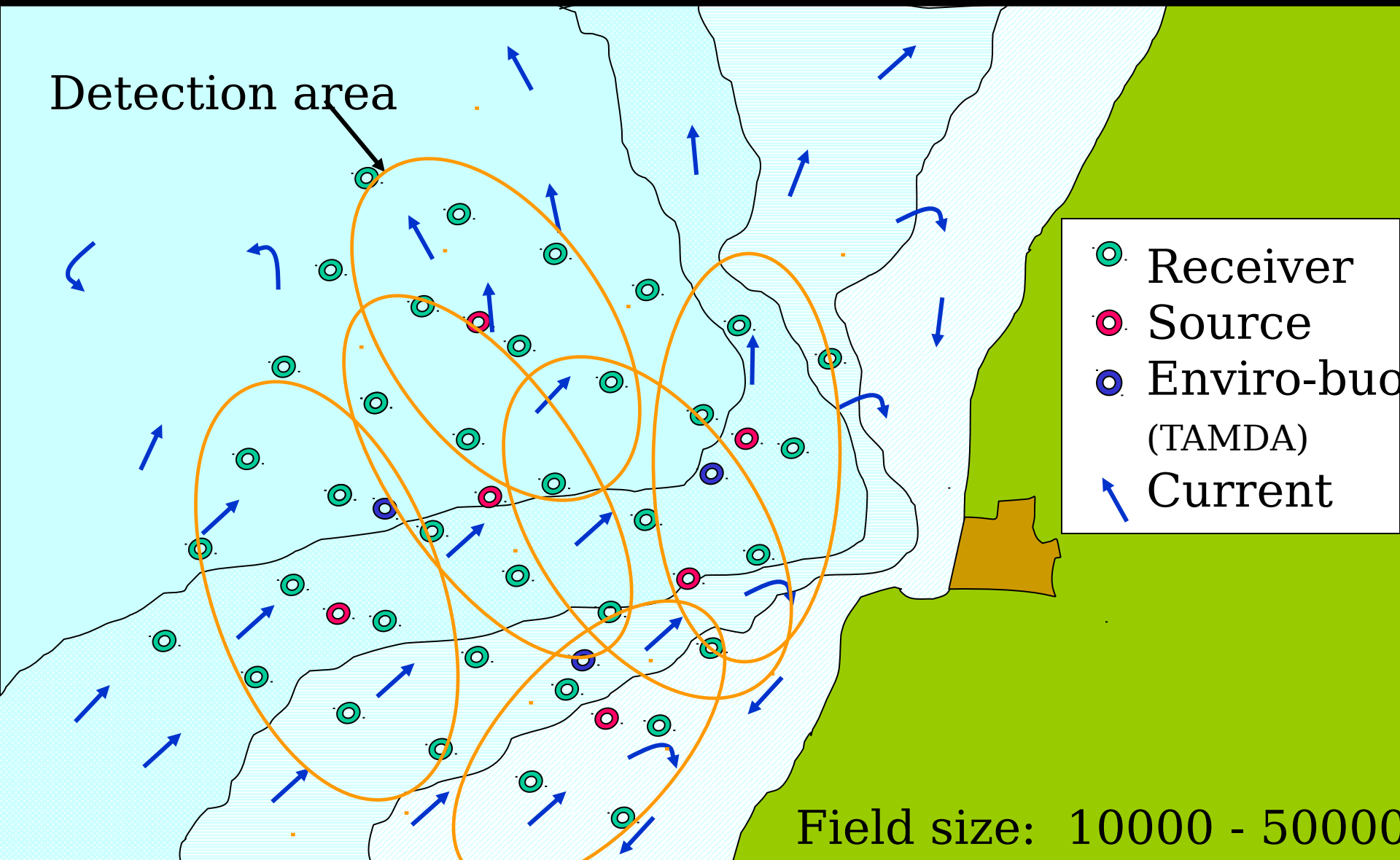
Applications

Prediction and Simulation of

- oil spills
- pollution dispersal
- drifting mines
- man over board
- Optimal Deployment of Drifting Acoustic Sensors (ODDAS)

Drifting System Concept

Acoustic Coverage T = 0



Drifting System Concept

Acoustic Coverage T = +1